## Waldorf Essentials

## A Journey Through Waldorf Math Class One through Class Five

by Melisa \& Erik Nielsen

Waldorf Essentials

# A Journey Through Waldorf Math Class One through Class Five 

Auriel's Light

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## Class One: Block One - Introduction to Mathematics

## Week 1: Roman Numerals

These riddles are intended to be given one at a time over the period of two weeks. Take your time with them. Consider drawing, painting or modeling from each one.

## The First Riddle

The first riddle goes like this:
I live in the sky
Up far away
I brighten the earth
I bring light to our days
And each night when the day is done
You will be sure I am the only one.
What am I?
(The Sun, and the Roman numeral I)
Your main lesson book might have a drawing of the sun or moon or your child. By this time your child should be able to write some short sentences, even something as simple as "Our sun is I." This may seem like a short lesson - it is meant to be. Toss bean bags in preparation for bean bag math, work on counting together, marching or skipping rope. Remember that it is natural for children learning to count to have trouble with the teens ( $13,14,15$, etc.) as they don't sound in English like the other numbers do. Some have found it helpful to teach them to count " 10 and 1, 10 and 2, 10 and 3 , etc." all the way to twenty when the numbers begin to sound like the first then again ( $21,22,23$, etc.) These sorts of activities will fill in the gaps as they work up to harder math that will come once the processes are introduced in later lessons. Also, remember hopscotch? This is a good time to introduce it! It is a counting activity.


## Class One: Block Two - Mathematics

Were you with us for kindergarten? Do you know our friend Super Sam? Super Sam is a sweet little gnome that used to live in the forest in a tiny little house shaped like a mushroom. Sam loves animals and has many friends. Last year Super Sam became a Number Gnome. It is such an honor to become a Number Gnome and work in the number kingdom with the other gnomes helping children all over the world learn about numbers. Super Sam was missing his family, so he decided to draw some pictures for them and write to them about his adventures since moving to the Kingdom of Numbers. These are his letters home.

Parents, we are going to start the block working through the Quality of Numbers. We started earlier with Roman numerals through riddles, now we are going to take that farther and introduce the Arabic numbers. Many children are counting on their own at this point, but even if your child is a budding mathematician, do not skip these lessons. In these lessons we will also bring some shapes to life. See how many you can find organically in your daily life. Shapes are a fun way to discover the qualities of numbers. We also snuck a short fairy tale into these lessons. As we move through the block we will also introduce you and your child to some of Super Sam's friends, the Math Gnomes and their work with the four processes.

## Week 1 - Review Roman numerals and Introduce Number Qualities

Super Sam was missing his family and friends back in the forest. He loved being a Number Gnome but his heart missed his little mushroom house, his friend Lola the turtle and all the animals that lived in his barn. When Sam writes home, he tells his family all about the numbers and their qualities that he is learning about in the Kingdom of Numbers.

Dear Mother, Father and Baby Bird,
I hope you are well. I have only been gone a short time but I miss you already. I decided I would write to you about all the different things I am learning about numbers and some of their stories. Here is a poem I wrote:

One is the Sun that shines above, Two are the parents that I love, My parents and my sister are three,
Four is our family, so precious to me,
Five are the stars shining over my head,
Six is the honeycomb sweetening my bread,
Seven is the rainbow that crosses the sky,
Eight are the friends that can count high.
Nine are squirrels that share our way,
Ten are the hours I work each day.
Baby Bird, you would love the squirrels! They are so silly. We are counting jewels and then keep bringing us nuts.
I love and miss you all,
Super Sam
Inspired by the original poem "One is the Sun" in The Waldorf Book of Poetry.

## The Numbers 1, One, I and 2, Two, II

The number one (in Arabic) looks a lot like the Roman numeral one, but not always. You may want to show the different ways people like to make the number one: 1 or 1 or 1 - all of them are the same number. Also introduce your child to the written word ONE. You could do a simple drawing like you did together for the Roman infroduction or do a drawing or painting from the story. What things in your life are there only ONE of? Head, house, maybe car? Practice writing ONE.

One is the Sun who shines above.


What things are there TWO of? Hands, eyes, legs, parents? Practice writing TWO.
Two are the parents that I love.


## Super Sam and The Math Gnomes - Introducing the Four Processes and Whole to Parts Math

Steiner worked with the concept of "whole to parts" with just about everything. We often only see the world as parts but we know we are much more than that. I am not just an arm or just a brain. A flower isn't just the bloom. A dog isn't just a tail. When we are talking about people or parts of nature we can easily see the whole, but what about in our world? Often the world and the modern media shows us only one perspective and that one perspective can be all people see if they don't insist on seeing the whole. Of course in this lesson we will only focus on the concept of whole to parts as it relates to mathematics, but as your child grows through this curriculum, bit by bit you will see that we are always working from the whole.

When working with this concept for math, the best way to describe it is like this:
While $3 \times 4$ is 12,12 is more than $3 \times 4.12$ is $6 \times 2,2 \times 6,4 \times 3,12 \times 1,6+6$, and so on.
So how is that translated practically for a child? You can ask a child the open ended question of "what is 12 ?" and allow many answers because there are many answers, allowing them to see the big picture of all that 12 really is, rather than only giving them a small representation of 12 and telling them later "and by the way, XYZ is also 12. ." Discovering numbers for a child is an experience that comes very much from their core of how they understand the world around them. They see the family as a whole; they rarely see just mom or just dad when they are young (before seven years.) Even if you have a single parent household, children see their family as a WHOLE, including grandparents, friends, etc. and so to bring them only pieces goes very much against how they see the world.

## Steiner says in Teaching Arithmetic:

"The living thing is always a whole and must be presented as a whole first of all. It is wrong for children to have to put together a whole out of its parts, when they should be taught to look first at the whole and divide this whole into its parts; get them first to look at the whole and then divide it and split it up; this is the right path to a living conception."

This is very foreign to most of us and how we were taught math. Many of us were counting and trying to memorize numbers and their abstract symbolism far before we could really understand it - I am amazed at children's television programming and so called "math help" for preschoolers and babies! Babies need to be babies, not counting machines! Most of us were taught to add and subtract first only to be bombarded with multiplication and division in later grades. Steiner's math concepts have children learning all four math processes on the same day! Steiner believed that introducing them all at the same time would allow for true freedom in thought as children grew into adults. Think about that for a moment...there is freedom in knowing that there is more than one way to derive an answer. This is true for all things, not just math. He discusses synthesizing verses analyzing - when we synthesize something, we have to add something together working up from the parts, but when we take time to analyze something we can separate it out or divide it into parts. Steiner believed that thinking had these two major components (synthesis and analysis) and that children by nature will choose to analyze things. With an education system that forces synthesis over analysis in these early years, he believed it would have strong consequences later in life - spiritual ones! Now whether or not you believe it, take some time to look at the world around you. Materialism abounds from the thought pattern that we must add more and more to become whole. How many of us have gone on spiritual journeys as an adult to become whole only to find out we already were whole and our ego just didn't know it? How much time would we have saved if we could have seen ourselves as whole? When we teach a child they are whole from day one, then we give them a great gift - a gift they can share with others. Much of Steiner's work sounds a lot like today's work in quantum physics, but this shouldn't be surprising...remember, truth is everywhere.

Now of course there are plenty of instances when synthesis is necessary and appropriate, but teaching from an analytical standpoint first allows the child the freedom to see what method will work best to solve each of life's challenges.

In The Renewal of Education, Steiner has this wisdom to share:
"If I have to add two and five and three in order to find the total, I am not free, for the answer is fixed by an underlying law. But if I begin with the number ten, I can view it as consisting of nine and one or five and five; or I can

The king counted out Divide's bag and sure enough, his gnomes came through once again. There were 10 jewels in her bag. He bid them a good day and called for the treasury gnomes to come and take away the jewels of the day.


## The Four Processes, Part 2 - Focus on Plus

Today you will want to reinforce the concepts learned in yesterday's lesson. There are so many ways to do this! I like to give the gnomes a chance to go for a walk; some days they go together and some days not. I will give you a couple of scenarios in the following lessons and you can always make up more if you need to. I think you will find that most children want to have more and more gnome stories, so get ready!

Today let's have Plus go alone. Your main lesson page may look something like our picture. Notice I added some writing to it as well as some math problems. I would write it all in your main lesson book; they can do a two page spread with the verse on one side and the math problems on the other. I would include some practice as well. The verses come from Harrer's book Math Lessons for Elementary Grades and she adapted them from verses written by Margaret Peckham.

Now you may look at this lesson and notice it isn't whole to parts! You might be saying, "Melisa, you told me that Steiner says it has to be whole to parts!" No worries! We worked from the whole (48) down to the parts with yesterday's work, but today we are dealing with Plus. By his very nature he is a greedy little synthesizer! He adds to his materialism constantly; he is a perfect example of what Steiner meant. Balance out today's work with whole to parts practice tomorrow! Some further writing for today might be to write the names associated with Plus. He is also known as Addition, but usually only his mother calls him that ("Plus Addition, get in here!") Point out that when Plus adds things together they are called "sums." You don't have to worry too much about making them memorize it just yet; I do like to make colorful signs for the schoolroom space, using recycled water color paintings for backgrounds and writing different "rules" as we come across them. It does help to have them write it, too.

When Plus adds things together they are called sums.

## Class Two: Block One - Mathematics: Time

Math in grade two is a continuation and building of skills learned in grade one and includes learning of time (including months, seasons, and reading the calendar) and money, column problems, and number patterns. Math blends into science when you cover seasons and time, the changes that occur throughout the year. This block may seem in some ways like a mainstream kindergarten review to some but it is much more. During this block on time you will explore things with your child that they can only now really understand. Their bodies are just now feeling the passage of time. If your child has special needs they may not be feeling this passage of time just yet, but the lessons are an important awakening for them.

These lessons will seem short in description, but I encourage you to really take the time with them. Begin discussing time. How do we measure time? Do we only measure it on a clock? A watch? How about a calendar? The moon? Are there different ways to measure time? Take some time this month to look at the moon each night and how it changes, talk about how people in ancient times (and even some now) used the moon as a time keeper. There are so many things you can consider during this block. I do not introduce new form drawings during math blocks but you are welcome to go back and practice ones you have done from earlier.

## Time: Months, Parts 1-4

In your lesson book this week you will focus on the months of the year. Have your child take the time to draw out the months in picture form. Some things that might be fun are to note everyone's birthday that you celebrate, favorite holidays, etc. What does January mean to you? If you ask my children they will tell you playing in the snow with friends and snowmen... if you ask me it would be dreaming about our next vacation! Everyone has a different idea of the month. One fun idea with these lessons would be to make a page for each month with a portion of the poem The Crowning of the Year by Juliet Compton-Burnett on each page, then have them copied into a calendar for family gifts.

Break each part up:
Part 1 - January, February, March
Part 2 - April, May, June
Part 3 - July, August, September
Part 4 - October, November, December
There are many sources for poetry and writing practice for the months of the year. I really like this excerpt from The Crowning of the Year by Juliet Compton-Burnett:

January brings the snow
But hark, the roots begin to grow.
February brings flowers of light
Petals three of snowdrops white.
March's blossoms - purple, gold, Six petals to the sun unfold.

April's here, gold trumpets sound And stars of white bedeck the ground.

May is the month of pink and white Apple, May, and Parsley light.

June brings flowers of rainbow hue Crimson, gold, and heavenly blue.

Flowers gay we still may see
Though dark the green on July's tree.

## Class Two: Block Two - Mathematics: Times Tables, Four Process Review, Number Patterns and Money

As you begin this lesson block, it would be good to review the four process math gnomes Plus, Minus, Times and Divide with your child. See if they can remember the story from last year. Also be sure to continue your daily movement with math through bean bag games, jumping rope, marching and games like hopscotch. As with other mathematics blocks, we will not introduce new form drawings.

## Times Table Clocks

So far your child has been learning the times tables either through movement games or by using the jewels to count things out. Today you are going to begin to translate that into something concrete they can carry with them to refer to while working in math. One thing that sets Waldorf apart from other methods is the notion of having them written for the child to refer to is somehow cheating. It is the act of continually working on the times tables through movement that gets them memorized. Tools to help them learn like the wheels we are going to make today are just helps; eventually they will not need them and find it quicker to just memorize. If you have worked whole to parts for your math facts then they should be more fluid to your child. Again remember that some special needs children may be a bit slower at this activity but stick with it; there is no shame in still marching out the times tables when they are 12 !

Today's lesson involves making fun times table circles like they keep on the walls in some of the Waldorf classrooms I have visited. Because space can be an issue, I opted to make smaller versions for my children from recycled paintings (can you tell we have a lot of paintings?!) and then we laminated them for durability, punched a hole in them and put them on a ring. See the companion CD for color versions of this project. You can also follow the example below. Do all the times tables through the 12 's starting with the ones your child knows easily through their skip counting which by now should be the 2 's, 3 's, 5 's and 10 's. The other tables should be added this year.



## Times Table Sheet

Today with your freshly made times table clocks, have your child do a regular table that you would normally find in a school setting. This will help with their spatial and relational skills as they go up and down, back and forth to insert their answers. If your child is intimidated by a completely blank table then help them out with a few answers; encourage them to do as many from memory as they can. The table will help them see patterns in math that we will cover later in this block. One thing they might do in a Waldorf classroom and it would be totally appropriate here would be to draw it out on fabric in crayon. Once you have them all you can iron it for a permanent piece that they can carry with them. We have also just used the first math block each year to create a new paper one.

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  | 14 |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  | 88 |  |
| 9 |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |  |  |  |  |  |  |
| 12 |  |  |  |  |  |  |  |  |  |  |  |  |

## Class Three: Block One - Mathematics Review

This short block is best worked at the beginning of the school year. It is especially helpful if you are new to Waldorf or didn't use Waldorf Essentials for Classes One and Two. We will cover concepts in review fairly quickly, one concept per day. Please feel free to take longer if needed. Conversely, if your child wants to go faster through these that is also fine; just make sure there are no gaps. In addition to the mathematics blocks we have listed here in the curriculum, you are encouraged to organically work practice into your days. Play games with a mathematics focus, practice times tables aloud while in the car, budget a trip together, etc.

## Review the Months

For today's lesson, review the sequence of the standard months by using this poem for writing practice:
Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save February, with twenty-eight days clear,
And twenty-nine each leap year.
To work on some drawing practice, have your child pick their favorite memory from the past year and illustrate it.


## Review Time

Many children have a pretty good concept of the passage of time by now, but it is still a good idea to review it. Review how many seconds in a minute, minutes in an hour, etc. Perhaps even make a sign for your school space if you don't already have one from last year.

## Time: Farmer Jack's Day

Today we will visit Farmer Jack. For main lesson drawings, you could have your child draw from the story with clocks next to each one to illustrate the time. At 6:00 a.m. Jack greets the day. He eats some hot cereal, then he's on his way. Jack milks the goat at 6:45 a.m. and feeds the hens at 7:00 a.m. He works the fields until lunchtime and comes home sometime around 11:00 a.m. At 1:30 p.m. he gathers hay and feeds it to the cows. At 3:15 p.m. he takes a break. His day is almost through, and the dogs and cats gather round to join in relaxing, too. At 4:10 p.m. Jack walks the farm and gathers the animals in, for evening comes around 5:30 p.m. and they need to settle in. It's dinner time for farmer Jack at 7:00 p.m. on the dot. What a day he's had; what fun it's been! Would you like to join him? Or not? For further writing and math practice: Farmer Jack has 10 hens. He also has 1 milking goat. Jack has fields full of corn and spelt. Jack has four cows. Five dogs and two cats live on Jack's farm. How many animals does Jack have?


## Class Three: Block Two - Mathematics

In this block will be discussing Biblical measure and relating it to how we measure things today. As we work with dry and liquid measure, be sure to get some baking time in together. Weeks three and four will bring in some geometry via area and perimeter as well as square, cubed, and prime numbers. Please take plenty of time to review all of the concepts.


## Block Two - Week One - Dry Measure

## Dry Measure Part 1

Today you will begin dry measure or mass. Let's consider a farming family that is preparing things to sell at the village market. The children in the family do the baking with their mother and need to figure out how much grain and sugar they need for their baked goods. It would be a good idea to have a small scale on hand if you can. A kitchen scale or postage scale will do; an old fashioned scale or even child's play scale would be great as well. We will tie this lesson to their other studies by using biblical measures. Scripture tells us that both David and Ezekiel attempted to standardize weights but neither could achieve complete uniformity. Some people tried to be unfair by having two sets of weights, an honest set and one that was off; it led to commoners carrying their own weights to see if they were being cheated. This is said to have upset some of the prophets because it was a sign of the poor spiritual conditions at the time.

Let's begin with the basics of weight measurement in ancient times, the shekel. It is uncertain where the practice of using grain as a weight measurement began, whether it be with the Egyptians or the Babylonians, but it is easy to see that it could be a fairly honest standard since grain should weigh about the same.

Using grain, wheat being the standard, but rice will do if you don't have any wheat, practice counting out what a shekel would be and then using the conversions for larger measure; take the time to measure things around your home.

In the main lesson book, consider a drawing like this one. A great verse for cursive practice relating to scales would be Proverbs 11:1 "The Lord abhors dishonest scales, but honest weights are his delight."

$$
\begin{aligned}
& 60 \text { grains }=1 \text { shekel } \\
& 60 \text { shekels } 1 \text { mina } \\
& 60 \text { minas }=1 \text { talent }
\end{aligned}
$$



It may be worth noting that early coins were representative of these weights, also. The Roman weights varied. Going back to the uncia, as the standard for weight also - remember it means $1 / 12$, but in this case $1 / 12$ of what? Certainly not the foot! The libra pondo, or pound, was used by the Romans as the standard for mass, but it again was hard to standardize. Interesting bit of trivia, though: we often see "lb"as the abbreviation for pound; this refers to the original term libra pondo.

Different kings tried to standardize throughout history, and trouble always arose when converting mass to liquid measure; this is a future lesson.

Through many years of arguing on the part of many men who all wanted to be right, we have these current measurements for mass:

- 16 ounces to a pound
- 2,000 pounds to a ton; however, a long ton is 2,240 pounds

Dry measure can also be measured as follows:

- 2 pints in 1 quart
- 4 quarts in a gallon
- 2 gallons in a peck
- 4 pecks in a bushel


## Class Three: Block Three - Mathematics

As we take on the last mathematics block of Class Three, I want to encourage you to practice often; in fact, you will notice that there is an abundance of practice time built into each of these four weeks. If your child needs more time please take it; if they are comfortable and ready to move on then do so.

Moving forward, continue to practice time tables orally in your morning gatherings until they are memorized. We aren't doing a lot of daily mathematics practice beyond the morning gathering until Class Four, and even then it is only a few problems each day. We want them to have a healthy relationship with mathematics, so if at any point you need to take a break, then do so. In Class Six and above, we will aim for about an hour a week of mathematics outside of the math blocks.

## Block Three - Week One - Place Value

This week we will begin with place value in preparation to introduce carrying and borrowing. To do this, we'll revisit our friends, the math gnomes. In preparation for these lessons, I like to have on hand flat marbles in bright rainbow colors; these can be found at most craft stores. The rainbow colors will represent the different types of jewels brought to the king each day.

## A Very Colorful Problem by Melisa Nielsen

As you remember from Class One, when our gnomes are done gathering their jewels for the day they take them to the king so they can be counted and used in the kingdom. Our four friends, Plus, Minus, Times and Divide never thought to ask what happened to the jewels next, so imagine their surprise at the end of one week when they saw the king upset about his treasury room. The treasury room was a place where only King Equals and the treasury gnome were allowed to go. The math gnomes from the kingdom each took a turn being the treasurer.
"I need your help," said the king. "The treasury gnome has been sick for a week and things are piling up in his work space. If you can help by counting out the jewels then the sacks of jewels can start to get sorted to their proper places."

The gnomes walked into the treasury room where there were piles and piles of jewels.
If the king received 48 jewels per day, 6 days per week, how many jewels are there? What if there were two or three weeks' worth?

The gnomes noticed on the wall a set of seven shelves, all color coded. Curious! The gnomes had no idea what this meant, so they decided to sit and take a break while trying to figure out a solution. While they sat, they noticed that there were some stones that were plentiful while others were not. This is something they had not paid attention to before, since most days they were only concerned with the collecting of jewels, not the kind. They noticed that the pretty dark purple amethysts were few, while the rubies were plentiful. They noticed that there were nearly the same number of some of the other stones. Divide had the idea to sort them by type and color, the topaz in one pile, the ambers in another, and the deep blue sapphires in another. Plus jumped at the opportunity to gather and count, so he began to count the rubies. Minus watched him trying to decide what to do when the piles began to spill over. He found that nine jewels fit in a pile well but 10 made the pile too full. It wasn't long before he realized that while dividing them up was helping bring order to the mess, it wasn't helping them clear the jewels or get them ready to leave the treasury. What could it be that the treasury gnome did with all those shelves? And how did he sort all of those jewels? Minus and Divide talked about the shelves while Times and Plus chatted away, counting out the jewels. Each time they put nine in a pile they would carry over and start a new pile (so they didn't have to hear Minus whine about them falling). Watching them and looking back at the shelves gave Divide an idea.
"What if these colored shelves are what the treasury gnome uses to sort out each type of stone? There seems to be a shelf for each kind we have here," said Divide.
"This is a good idea," said Minus. "I bet he does it so he can keep track of it all; let's try it."

They began to move the piles of colors to the shelf and noticed that at the end of each shelf there was a message. At the end of the red shelf was a note that said, " 9 rubies is 1 amber." Then they looked at the end of the orange shelf and it said," 9 pieces of amber is 1 topaz." Divide finally understood! She grabbed Minus and they looked at the rest of the shelves. They each had such messages on them, and they also noticed that at the end of the shelves there were color coded bags with the same message on it. As Plus and Times piled up the stones, Minus and Divide put them in the proper space, and each time there was a pile of 10 they moved them to a bag and put them in the proper pile. In no time they had all of the jewels sorted.

For practical work, take your colored jewels and have your child sort them so that there are only nine in each pile. We have small paint jars that we have labeled with the place value for each color: red ones, orange tens, yellow hundreds, green thousands, blue ten thousands, indigo hundred thousands, and purple millions. To practice what goes where, I give simple problems like 495,429 and ask how that would look in jewels. Of course it will take them a moment to understand that we move from right to left in math; this concept is hard for many children. Practice this for the day until they are comfortable with it, reminding them that there can only be nine in a pile. Ask what would happen if we added one more to the ones? Can your child easily see that no more can be added to the pile and that one more must be added to the next pile over?


## Place Value Practice

Now that the gnomes have had their time in the king's treasury, they want to make sure things never get out of hand again. Help the gnomes make a new chart for the treasury wall so that it is easy for the treasury gnome to have a sick day or a break now and then.

## Class Four - Mathematics Block One

Welcome to our first mathematics block of Class Four. Hopefully, you have been continuing with practice problems. We'll begin with some review and then go right into long multiplication and division.

I like to make a title page for the main lesson book.


## Block One - Mathematics Review Week One

Spend this week reviewing. Please do not underestimate the power of review. By this point your child should be getting fairly proficient at their times tables, at least through the 6's or 7's. Keep building. We like to play math games, toss bean bags, play math bingo and other fun games during this review week. We have two review main lesson page samples. Write the problems in their main lesson book and see what they can do without your help. These are concepts from Class 1 and up. The end of this chapter has story problems written for this week; you can work these orally or have them draw in their main lesson book from them. I also like to have them create a new times tables sheet. This "cheat sheet" is something they can keep all year as they are practicing proficiency at their multiplication. A larger review will follow Block Two.

If by chance your child is not at this place in their mathematics journey, do not fear! Go back to a place more comfortable for your child, that is the beauty of having all the grades here in one place.


## Block One - Mathematics Week Two

## Long Multiplication Part 1

Today's lesson will focus on multi-digit long multiplication. Be sure before you do this lesson that you have reviewed carrying with your child, as that skill will be crucial.

I like to have my child record this process in their main lesson book or even in a separate little book they can use for all the processes and sequences they will be learning from here on. A sample Math Sequences book is on the next page. The main lesson content is the same; teach from one and help them write the process in their book or you can write it for them in their processes book while they record it in their main lesson book.

## Class Four- Mathematics Block Two

Welcome to our second mathematics block! In this block we will introduce fractions and go more in depth with long multiplication and division. We will have much more fraction work in Class Five; for now we are just practicing and getting used to the terms.

## Block Two - Mathematics Week One

## Review All Known Concepts

Make today a review day. Hopefully you have been working on math concepts a bit each day so this should be easy. If you don't feel like you need the review then please walk forward to the next lessons. Look at all concepts learned so far and put together a main lesson page for your child. I find this to be the best "test" of where they are; it will give you a good idea of where you still need to go.

## Basic Fractions Introduction

Most children will already know basic fractions because you have likely been baking together. Take some time to get to the heart of what they know by testing them a bit. Since we can start with simple baking rules, take one cup (the whole). How many ways can we break it down to parts? Most of our kitchens have measuring cups to about $1 / 8$ cup.

By playing with the cups, it is easy to see that 1 cup contains in it $2-1 / 2$ cups, $4-1 / 4$ cups, $8-1 / 8$ cups. Now, what about that $1 / 3$ cup? That's an odd one; how many of those are in 1 cup?

I like the illustration of a dozen eggs (buy some brown, some white or if you have hens that lay different eggs) - while an egg is a whole unit alone, it is also part of a larger unit of 12 . Now these kids have been breaking down 12 since they were in first grade so they should quickly be able to understand this relationship. Fractions are about making equal pieces. Really from the day they learned to play with Divide, they were learning about fractions; we just never took it to them in that manner. Having both white and brown eggs out, agree that each egg is $1 / 12$ of the whole dozen. Now play around with the ratio of white to brown eggs. If 5 out of 12 were brown, how would you write that? Now what about 6 out of 12 ? Is there another way to say $6 / 12$ 's? What is $6 / 12$ 's? A half dozen? If $6 / 12$ equals half, then what does $3 / 12$ (half of the half) equal? You don't have to reduce the fraction consciously just yet, just get them thinking about it.

Here is a quick story to help illustrate this.
Remember our farmer from the last math block? Today we are going to talk about his daughter, Sally. Sally is responsible for the hens and making sure that each day all the eggs get collected. At the end of each week Sally helps her father put some of the eggs in cartons to take to the market to sell. Sally tends to many hens, but there are four that she loves the most because of the rich colors of eggs they lay. These hens are called Judy, JoJo, Emma, and Petunia. Judy lays eggs that are a rich caramel color, JoJo's are a deep brown, Emma's a dainty creamy tan and Petunia's are a beautiful blue green. When Sally was younger, the farmer let her choose the hens as pets, and Sally picked the hens with the prettiest eggs.

Four hens each lay one egg per day from Sunday to Saturday. How many eggs does Sally collect from her favorite hens?
$7 \times 4$
Sally is also responsible for 20 other hens, who are perfectly lovely but lay only brown and white eggs. Each of those hens also lays an egg each day. How many eggs does she collect from this group?
$7 \times 20$
Now when it is time to prepare the eggs for the market Sally loves to choose the eggs that will stay behind and feed their family for the week. Sally carefully chooses the best ones for the refrigerator egg basket. She chooses 4 from Judy, 2 from JoJo, 3 from Emma and 3 from Petunia. What fraction of the whole (12) are the eggs from each hen?

We want to play with this concept in a few different ways so they are used to associating them tangibly as well as written, so we've included two different drawings for you to choose the one best for your child.


## Class Five: Block One - Mathematics Review

Welcome to the first block of Class Five! As we begin the year, I like to start with a thorough mathematics review. This will give you a good baseline to see if there are any gaps in their knowledge. It will also get them primed for the first mathematics main lesson block that comes in Block Two. If your child is still challenged by these concepts, do not worry - we are looking for them to have progress over time. Keep practicing, use the math sequence book we built in Class Four, and remember that preserving the relationship with your child should be top priority. Take your time. If, on the other hand, your child is breezing through the concepts, feel free to go deeper with what they know. There are practice problems in the lessons as well as the back of the curriculum.

## Block One - Mathematics Review - Week One

While it is tempting to skip reviews, I highly recommend you squash any of those ideas. While there are several problems here, please do not feel like you need to do them all. We want to give you variety and enough for hungry students!

## Review Times Tables

By this point your child should have a good grasp on most of the times tables. Take some time today to have your child create a new times table "cheat sheet" for this year. In my experience most children tire of using the sheet and just commit them to memory, but just to be sure, practice them regularly.

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  | 14 |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  | 88 |  |
| 9 |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |  |  |  |  |  |  |
| 12 |  |  |  |  |  |  |  |  |  |  |  |  |

## Review Time, Money \& Long Division

Some review, including time, money and long division. You could take some time to split this lesson up and also put some drawings about it in your lesson book.

Let's consider a farmer who has much to do and limited time to do it. Once a week, he takes milk and eggs to market. Today he is taking 20 gallons of milk and 360 eggs. This week he has 10 pounds of fresh butter, two dresses and 5 pounds of wool to sell at the market as well. The market is 30 miles away. His truck travels best when he drives about 60 miles an hour for speed. The market opens at 5 a.m. and the farmer wants to be there when it opens.

How long will it take for him to get to the market?
If it takes him an hour to do the morning farming chores and 30 minutes to get dressed, what time will he need to get up in order to get to the market on time?

How many dozen eggs is he taking?
He expects to fetch:
$\$ 1.50$ per gallon of milk
50 cents per dozen eggs
$\$ 2.00$ per pounds of butter
$\$ 10.00$ for each dress
$\$ 5.00$ per pound of wool
If he sells everything for the amount he is seeking, how much will he have to spend?
He needs to purchase the following:

- 10 pounds of wheat (for flour)
- 5 pounds of sugar
- 20 canning jars
- Gas for his truck
- A gift for his wife's birthday

The farmer's wife needs the flour and sugar to make scones for an afternoon tea party and the canning jars to put batches of her fresh jam into. Her party begins at 3 p.m. The preparation time for her tea scones is about 40 minutes 20 minutes to grind the flour and 20 minutes to put the ingredients together. The scones cook for about 15 minutes. So she is ready in time for her party, what time will she need to begin preparing her recipe? What time will the farmer need to leave the market in order to be home with enough time for his wife to bake?

When the farmer arrives at the market he quickly sells his products and gets the price he wanted for everything except the wool; he could only get $\$ 3.00$ per pound of wool. How much does he have to spend?

He finds out that wheat is $\$ 2.00$ per pound, sugar is $\$ 1.00$ per pound, and the canning jars are $\$ 0.20$ each. What else was there? Oh yeah! He needs gas for his truck! Oh, and there was something else...he knew that he should have made a list... what was it? Oh good, you remembered - his wife's birthday gift!

Gas for the truck is $\$ 0.85$ a gallon and he needs 12 gallons to last him the week. (You'll have to help with the decimal since we haven't covered that yet.)

The farmer knows how much his wife enjoys tea. He finds a beautiful tea set for $\$ 20$ and some wonderful imported tea for $\$ 2.00$ per pound. He buys two pounds.

How much money does he arrive back home with? Oh wait! He forgot to take out money for tithing. Go back and look at how much money he got for his sales. $\$ 0.10$ from each dollar belongs to God. Now how much does he have left over?

You can allow your child to live a story like this very easily by letting them consciously observe life with you. Many times we try to shelter them from the costs of everyday living, but I find that they have a much better appreciation for our job if they can know and understand the stress involved with putting together a budget and sticking to it.

Working this scone recipe together would be a great review for measurement and also prime your child for the upcoming lessons on fractions. I have made these savory by omitting the sugar and upping the salt content a bit adding cheese and garlic makes wonderful garlic cheddar scones for a great dinner treat and they are fast to put together!

## Class Five: Block Two - Mathematics

The work done in Class Five will set your child up for success as they move forward with mathematics. They should fully understand fractions and decimals, giving them context for many common things in life - for instance, grocery shopping is much easier if you can quickly add dollars, building anything is simpler if you can quickly add inches to feet, and many other common, everyday concepts. While I don't use that long division every day, I do use my knowledge of decimals and fractions almost daily, so if this is an area that stresses you out, make sure you take the proper time to prepare these lessons. We should continually be giving a few problems each day. You can always pull the problems for the first week of Block One as well as any of the problems in the math blocks; you do not continually need new problems to work with. Also a few is four, not fourteen or twenty-four! These new concepts can be challenging for some, so making sure you take proper time for introduction is key and daily review of the previous lessons will also assist in helping them stick with your child.

I am often asked if a child who is still struggling with long division can move forward. My answer is that it depends. The questions I ask are these: Does your child have a sense for how the processes work and they are just having struggles with the sequencing of things? If so, this is often resolved with making a Math Sequences book that they can use as a template. We will be adding to the one made in Class Four. Take some time to see where the true challenge is with this child. Is it times tables recall? Is it slowing down to follow the directions in their sequencing book? Is reading still a challenge? These are all things we can work toward solving. Reach out to our team or come to office hours so we can help you make a plan.

I do think that fractions and decimals can be some of the hardest work in math, especially if we miss something and then try to move forward. When I look at my own math gaps and struggles I had as a child, it can easily be pinpointed back to one or two lessons where I must have been absent in school and could not catch up without teacher help. Teaching our children can heal these gaps in so many ways - you are learning so much by teaching them! Also, don't hesitate to ask for help! If your partner or friend is better at mathematics than you are, ask them to first help you. Get to feeling confident with presenting these problems to your child; you will be happy that you did. There is no shame ever in asking for help.

## Block Two - Mathematics - Week One

## Reducing Fractions \& Finding Equivalent Fractions, Part 1

Today's lesson will be reducing fractions and finding equivalent fractions; it is worth putting into your child's Math Sequences book. Begin with equivalent fractions - this is easiest to do with visuals. Try using this as a visual for your chalkboard and/or lesson book. Also, change it up a bit. Don't just stick to using $1 / 2$; also look at thirds, quarters, etc. Also, change the shapes up. Use circles, rectangles, etc. Today you'll use the visual and tomorrow we'll do some reducing without using the shapes.


What is $1 / 2 ?$
$\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \frac{4}{8} \quad \frac{8}{16}$

## Reducing Fractions \& Finding Equivalent Fractions, Part 2

Today we will reduce fractions to their lowest terms by finding common factors. Put this process into the Math Sequences book on the same page that you put the previous lesson's examples. Lessons on prime factoring from Class Four are useful; review them if needed. Teach them to find the greatest common factor or GCF in order to reduce a fraction. Remember, the GCF is when we find the number or factor that each part of the fraction has in common. Once you find the greatest common factor, then you divide both the numerator and the denominator to get the reduced fraction. It is helpful to note that sometimes the numerator can be the factor.

Here is an example:

$$
\begin{array}{r}
\frac{4}{12} \div 4=\frac{1}{3} \\
\frac{5}{20} \div 5=\frac{1}{4}
\end{array}
$$



Ask your child if they can tell you what they think the right way to write $4 / 3$ might be. Remind them that if $3 / 3=1$ then what could $4 / 3$ equal? Main lesson work could be to draw the bagels (or donuts!) illustrating how $4 / 3$ is 1 and $1 / 3$. See if they can also visualize a few others in this manner.

14/12
10/3
7/6
Once they have gained a visual, teach them a quicker way. Copy this example into their Math Sequences book. Plenty of practice should follow.

Turn 25/3 into a proper fraction without using a picture.

$$
\frac{8}{3} \text { Remainder } 1
$$

The 8 becomes the whole number and the 1 the numerator, the 3 the denominator.

The answer looks like this: $\quad 8 \frac{1}{3}$

## Class Five: Block Three - Mathematics

Today we will start our last mathematics block. Remember, you should be working with some problems daily even when not in a math block just to keep concepts fresh. There are problems to pull from in the back of the curriculum. This block will introduce decimals. Be sure to thoroughly prepare for this block. Work every problem before you bring it to your child. If you need more than one lesson day to practice then please take all you need so your child is comfortable.

## Block Three - Mathematics - Week One

## Introducing Decimals

Today you will introduce the decimal system. If necessary, do a quick review on common fractions before beginning these lessons.

Let's start from the beginning. The word decimal comes from the Greek word "deka" which means ten. The month of December was named because it was the tenth month of the Roman calendar. (Their year started in March.) Point out for your child other places where the root word "deka" appears. A decade is a period of ten years. The American currency is based on decimals - using multiples of 10 to make a dollar, a hundred dollars, etc.

It would be a good idea to review place value before starting this lesson as a new place value will be introduced today. It would also be a good idea to make a new place value chart for your school space and their Math Sequences book if you are keeping one.


The easiest way for me to explain an introduction to decimals is to use the American currency system; even if you are not an American, it is pretty straightforward. If you aren't in America and have a similar currency system then feel free to exchange it for yours. Very simply, there are 100 pennies in a dollar. Even if you get lost on what nickels, dimes and quarters are worth, just remembering that there are 100 pennies in a dollar will save you! So if we also remember that the more dollars you have the better off you will be, then you can easily understand decimals to the hundredths.

Using the 100 pennies to the dollar concept, talk to your child about which they'd rather have...which is bigger?

## 7.0 . 70.07

Of course the answer should be 7.0 ! I would much rather have $\$ 7.00$ than 70 cents or 7 pennies.
How about 6.58 . 65 . 06 ? It is important to remember that the farther to the right you get, the less money or the smaller the fraction of 100 you have.

It is best to get your child as comfortable with decimals in this lesson as possible - in order to do that, go back to common fractions and convert them to decimals.

Again, thinking about money is helpful:

- $1 / 10$ of a dollar $=10 ¢$ or .10
- $2 / 10$ of a dollar = $\qquad$
- $3 / 10$ of a dollar $=$ $\qquad$
- $4 / 10$ of a dollar $=$ $\qquad$
After you've thought that one through, try going further:
- $1 / 100$ of a dollar $=1 申$ or .01
- $10 / 100$ of a dollar $=$ $\qquad$
- 15/100 of a dollar = $\qquad$
- $35 / 100$ of a dollar $=$ $\qquad$
Now think about $1 / 2$ of a dollar or $50 \varnothing$, which is noted in decimals as .50 .
Which is easier to think about, $1 / 4$ of a dollar or .25 ? Once you can get comfortable thinking about money in decimals the rest will make much, much more sense. Take some time to add and subtract using the concept of pennies to dollars.
- . $50+.75=$ $\qquad$
-. $35+.45=$ $\qquad$
-. $77+.23=$
- . $86+1.28=$ $\qquad$
- $5.24+.86=$ $\qquad$
Once they are comfortable with adding money, chances are they will be able to quickly visualize the answer. You want to continue this with the lessons to follow. Practice as often as you can when shopping or counting out allowance. Making change is also a great way to continue practicing. For today, just work on oral practice and the place value chart in the main lesson book or in their Math Sequences book.


## Resources \& Supplies

As always, my aim is to help you save money whenever possible. If you take really good care of your supplies, they will take care of you. I still have crayons and pencils that are nearly ten years old! Remember that wonderful supplies alone will not make beautiful lessons; planning and being prepared should be your first priority, supplies second.

Helpful tools in your mathematics journey:

- Classes 1-3: Math jewels, stones, shells or some sort of counting manipulative.
- Block \& stick crayons for main lesson drawings.
- Lyra colored pencils for main lesson drawings.
- Lead pencils for writing with erasers, most important for classes 3 and up.
- Main lesson or sketch book to put their lessons in.


## Waldorf Curriculum, Major Themes Grades 1 to 6

There is a more complete chart on our website in the Resources section.

| Grade 1 <br> History, literature \& LA <br> - Learning to read and write with the help of the fairy tales as a base for stories <br> Mathematics <br> - Counting, introduction to the four processes through stories Geography \& science <br> - Home surroundings <br> Languages -foreign languages can be taught, through song and culture, no tapes or TV <br> Art \& music <br> - Painting, drawing \& modeling from lesson work <br> - Knitting simple projects <br> - Penny whistle or recorder \& singing | Grade 2 <br> History, literature \& LA <br> - Fables \& saint stories to continue the reading journey, local folklore <br> Mathematics <br> - Continue with times tables, time \& money, extending skills with the four processes <br> Geography \& sciences <br> - More home study surroundings, observing animals from their fables work <br> Continue language study <br> Art \& music <br> - Continue painting, drawing \& modeling from lesson work <br> - Begin knitting more complicated projects <br> - Continue with instrument \& singing | Grade 3 <br> History, literature \& LA <br> - Biblical stories as part of history <br> - Full introduction to parts of speech, begin sentence structure <br> Mathematics <br> - Continue times tables, begin measurement, place value, carrying \& borrowing <br> Geography \& sciences <br> - Cycles of the year <br> - Farming, homes and dwellings, clothing, etc. <br> Continue language study <br> Art \& music <br> - Continue painting, drawing \& modeling <br> - Begin crocheting <br> - Continue with instrument \& singing |
| :---: | :---: | :---: |
| Grade 4 <br> History, literature \& LA <br> - Stories from Norse myth, local history <br> - Poetry, deepen grammar work, spelling <br> Mathematics <br> - Long divisions/multiplication, fractions, averaging, proofs, factoring <br> Geography \& science <br> - Zoology, man \& animal and their relationships <br> - Local geography, being aware of local plant life <br> Continue language study <br> Art \& music <br> - Continue painting, drawing \& modeling, more complex <br> - Sewing, embroidery, cross stitch <br> - Continue with blowing instrument \& singing <br> - Orchestra or string instrument | Grade 5 <br> History, literature \& LA <br> - Stories from ancient cultures \& history, India, Persia, Mesopotamia, Egypt, Greece, up to Alexander the Great <br> - Writing longer compositions Mathematics <br> - Geometry as part of drawing <br> - Decimals, fractions, metric system <br> Geography \& science <br> - Botany, plant and earth relationship <br> - Zoology, a continuation <br> - Geography of your own country <br> Continue language study <br> Art \& music <br> - Continue painting, drawing \& modeling, more complex <br> - Four needle knitting <br> - Continue with blowing instrument \& singing <br> - Orchestra or string instrument | Grade 6 <br> History, literature \& LA <br> - Rome, Life of Christ \& Muhammad, Medieval society <br> - Writing is more in depth Mathematics <br> - Geometry <br> - Business math, pre-algebra Geography \& science <br> - Geography of Canada, Latin America, \& countries being studied <br> - Physics, geology, more botany Continue language study Art \& music <br> - Continue painting, drawing \& modeling, more complex <br> - Sewing <br> - Continue with blowing instrument \& singing <br> - Orchestra or string instrument |



